$\int_a^b f(x)\,dx \longrightarrow \lim_{n\to\infty} \sum_{i=1}^{n} f(a+\Delta x i)\cdot \Delta x$ $\quad \Delta x = \frac{b-a}{n}$

$x = a + \Delta x i$

DEFINITE INTEGRAL AS A RIEMANN SUM

IN-CLASS SAMPLE PROBLEMS

Rewrite the definite integral as a limit of a Riemann Sum

Ex. 1 $\int_0^{\pi} \sin x\,dx$

$\Delta x = \frac{\pi}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(0+\frac{\pi}{n}i\right)\cdot\frac{\pi}{n}$

Ex. 2 $\int_2^6 \frac{1}{5}x^2\,dx$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{5}\left(2+\frac{4i}{n}\right)\cdot\frac{4}{n}$

Ex. 3 $\int_1^e \ln x\,dx$

$\Delta x = \frac{e-1}{n}$

$x = 1+\frac{(e-1)}{n}i$

$\lim_{n\to\infty} \sum_{i=1}^{n} \ln\left(1+\frac{(e-1)}{n}i\right)\cdot\frac{e-1}{n}$

Rewrite the limit of a Riemann sum as a definite integral

Ex. 4 $\lim_{n\to\infty} \sum_{i=1}^{n} \ln\left(2+\frac{5i}{n}\right)\cdot\frac{5}{n}$

$a=2 \quad \Delta x = \frac{5}{n} \quad b-2=5$

$b=7$

$\int_2^7 \ln x\,dx$

Ex. 5 $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4+\frac{2i}{n}}\cdot\frac{2}{n}$

$a=4$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$b=6$

$\int_4^6 \sqrt{x}\,dx$

Ex. 6 $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{12k}{n}\cos\left(1+\frac{4k}{n}\right)\cdot\frac{4}{n}$

$\int_1^3 (3x-3)\cos x\,dx$

$x = 1+\frac{4k}{n}$

$3x = 3+\frac{12k}{n}$

$3x-3 = \frac{12k}{n}$

Ex. 7 $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{10i}{n}\left(\sqrt{1+\frac{5i}{n}}\right)\cdot\frac{5}{n}$

$\int_1^6 (2x-2)\cdot\sqrt{x}\,dx$

AP Multiple Choice

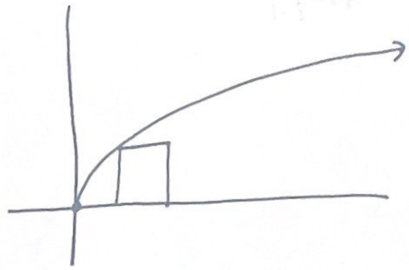
The definite integral $\int_0^4 \sqrt{x}\, dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}\, dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}\, dx < R$

(C) $R < \int_0^4 \sqrt{x}\, dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}\, dx < L$



f(x) is inc → L is under
R is over

f(x) is concave down → T is under but > L

$L < T < \int_0^4 \sqrt{x}\, dx < R$